**Problem 1: Basic Vector Operations** (12 points, 4 points/each)

Let two vectors and , answer the following equations:

1. Calculate the norm of a and b.
2. Calculate the Euclidean distance between a and b (i.e. norm of ).
3. Are a and b orthogonal? State you reason.

They are orthogonal, because and .

**Problem 2: Basic Matrix Operations** (40 points, 4 points/each)

Suppose , answer the following questions:

1. Calculate and .

From , we have that

1. The Rank of A is?

We can convert the matrix into a stepped matrix to solve the rank, but notice that , so .

1. The trace of A is?
2. Calculate .
3. Is an orthogonal matrix? State your reason.

A is not an orthogonal matrix, because .

1. Calculate all the eigenvalue λ and corresponding eigenvectors of A.

Calculating the eigenvalues is equivalent to finding all such that the eigen equation have nontrivial solutions. By the inverse matrix theorem, we only need to find all such that is irreversible, so

Next, let , simplify the augmented matrix of by row transformation.

When ,. When ,or

1. Diagonalize the matrix A.

Matrix A can be diagonalized, if there is an invertible matrix P and diagonal matrix D, such that .

The diagonal matrix D is consists of eigenvalues.

The invertible matrix P is consists of eigen vectors.

Therefor,

1. Calculate the norm and the Frobenius norm (i.e. norm) .
2. Calculate the nuclear norm and the spectral norm .

For the nuclear norm , according to its definition, that is, the sum of singular value, we have

For the spectral norm , according to its definition, that is, the max singular value ,we have

**Problem 3: Linear Equations** (48 points, 4 points/each)

Please give some proper steps to show how you get the answer.

Let and

Answer the following questions:

1. Solve the linear equations 1 (6 points) (6 points)

By basic elimination, we can get simply

1. Write it into matrix form(i.e. ) and we will use the same A and b in the following questions.
2. The Rank of A is?

There are many ways to obtain , for example, we can show that (see below) or calculate the number of linearly independent rows by elementary transformation.

1. Calculate and .

From , we have that

1. Use (4) to solve the linear equations

First of all, write the augmented matrix

Then, transform A into a reduced row echelon matrix.

Therefore, we have

1. Calculate the inner product and outer product of x and b.(i.e. and )
2. Calculate the, and norm of b
3. Suppose , calculate , .
4. We add one linear equation into linear equations above. Write it into matrix form(i.e.)
5. The rank of is?

Transform into a row echelon matrix to find the number of linearly independent rows.

Therefore, .

1. Could these linear equations be solved? State reasons.

It is solvable and has infinitely many solutions, because .